The Modal Interpretation of Algebraic Quantum Field Theory

Rob Clifton

Department of Philosophy, 1001 Cathedral of Learning,
University of Pittsburgh, Pittsburgh, PA 15260
(E-mail: rclifton+@pitt.edu)

Abstract

In a recent Letter, Dieks [1] has proposed a way to implement the modal interpretation of quantum theory in algebraic quantum field theory. We show that his proposal fails to yield a well-defined prescription for which observables in a local spacetime region possess definite values. On the other hand, we demonstrate that there *is* a well-defined and *unique* way of extending the modal interpretation to the local algebras of quantum field theory. This extension, however, faces a potentially serious difficulty in connection with ergodic states of a field.

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1 The Modal Interpretation of Nonrelativistic Quantum Theory

The modal interpretation is actually a family of interpretations sharing the feature that a system's density operator constrains the possibilities for assigning definite values to its observables [2]–[7]. We start with a brief review

of the basic idea behind these interpretations from an algebraic point of view [8]–[10].

Consider a 'universe' comprised of a quantum system U, with finitely many degrees of freedom, represented by a Hilbert space $\mathsf{H}_U = \bigotimes_i \mathsf{H}_i$. At any given time, U will occupy a pure vector state $x \in \mathsf{H}_U$ that determines a reduced density operator D_S on the Hilbert space $\mathsf{H}_S = \bigotimes_{i \in S} \mathsf{H}_i$ of any subsystem $S \subseteq U$. Let $\mathcal{B}(\mathsf{H}_S)$ denote the algebra of all bounded operators on H_S, and for any single operator or family of operators $T \subseteq \mathcal{B}(\mathsf{H}_S)$, let T' denote its commutant (i.e., all operators on H_S that commute with those in T). Let P_S denote the projection onto the range of D_S, and consider the subalgebra of $\mathcal{B}(\mathsf{H}_S)$ given by the direct sum

$$\mathcal{M}_S \equiv P_S^\perp \mathcal{B}(\mathsf{H}_S) P_S^\perp + D_S'' P_S. \tag{1}$$

When S is not entangled with its environment $\overline{S}$ (represented by $\mathsf{H}_{\overline{S}} = \bigotimes_{i \notin S} \mathsf{H}_i$), D_S will itself be a pure state, induced by a unit vector $y \in \mathsf{H}_S$. In this case, $\mathcal{M}_S$ consists of all operators with y as an eigenvector — the self-adjoint members of which are taken, in orthodox quantum theory, to be the observables of S with definite values. On the other hand, when there *is* entanglement and D_S is mixed — in particular, in the extreme case $P_S = I$ (where essentially every vector state $y \in \mathsf{H}_S$ is a component of the mixture) — then $\mathcal{M}_S = D_S''$, and $\mathcal{M}_S$ consists simply of all functions of D_S. In this case, orthodox quantum theory has nothing to say about the properties of S. Thus, when S is Schrödinger's cat entangled with some potentially cat killing device $\overline{S}$, we get the infamous measurement problem.

In non-atomic versions of the modal interpretation ([3], [11]–[13]), there is no preferred partition of the universe into subsystems. Any particular subsystem $S \subseteq U$ is taken to have definite values for all the self-adjoint operators that lie in $\mathcal{M}_S$, and this applies *whether or not* D_S is pure. The values of these observables are taken to be distributed according to the usual Born rule. Thus, the expectation of any observable $A \in \mathcal{M}_S$ is $\mathrm{Tr}(D_S A)$, and the probability that A possesses some particular value a_j is $\mathrm{Tr}(D_S P^j)$, where $P^j \in \mathcal{M}_S$ is the corresponding eigenprojection of A. As in orthodox quantum theory, *which* precise value for A occurs on a given occasion (from amongst those with nonzero probability in state D_S) is not fixed by the interpretation. However, the occurrence of a value does not require that it be 'measured'. And, unlike orthodox quantum theory, no miraculous collapse is needed to solve the measurement problem. Instead, after a typical

unitary 'measurement' interaction between two parts of the universe — O the 'measured' system, and A the apparatus — decoherence induced by A's coupling to the environment $\overline{O \cup A}$ will force the density operator D_A of the apparatus to diagonalize in a basis extremely close to one which diagonalizes the pointer observable of A [14]. Thus, after the measurement, the definite-valued observables in $\mathcal{M}_A$ will be such that the pointer points!

In atomic versions of the modal interpretation ([2], [15]–[17]), one does not tell a separate story about the definite-valued observables for each subsystem $S \subseteq U$. Rather, S is taken to inherit properties from those of its atomic components, represented by the individual Hilbert spaces in $\bigotimes_{i \in S} \mathsf{H}_i$. In the approach favoured by Dieks [16] (cf. [15]), each atomic system i possesses definite values for all the observables in $\mathcal{M}_i$, as determined by the corresponding atomic density operator D_i in accordance with (1). The definite-valued observables of S are then built up by embedding each $\mathcal{M}_i$ in $\mathcal{B}(\mathsf{H}_S)$ (via tensoring it with the identity on $\mathsf{H}_{\overline{S}}$), and taking the von Neumann subalgebra of $\mathcal{B}(\mathsf{H}_S)$ generated by all these embeddings. The definite properties of S will therefore include all projections $\bigotimes_{i \in S} P_i^{j(i)}$ that are tensor products of the spectral projections of the individual atomic density operators, and their joint probabilities are again taken to be given by the usual Born rule $\text{Tr}[D_S(\bigotimes_{i \in S} P_i^{j(i)})]$. In the presence of decoherence, the expectation is that atomic versions of the modal interpretation can yield essentially the same resolution of the measurement problem as do non-atomic versions [16, 18].

In both versions of the modal interpretation, the observables with definite values must change over time as a function of the (generally, non-unitary) evolution of the reduced density operators of the systems involved. In principle, this evolution can be determined from the (unitary) Schrödinger evolution of the universal state vector $x \in \mathsf{H}_U$ [19]. But the evolution of the precise values of the definite-valued observables themselves is not determined by the Schrödinger equation. Various more or less natural proposals have been made for 'completing' the modal interpretation with a dynamics for values ([15],[20]–[22]). Unfortunately, it has been shown that the most natural proposals for a dynamics, particularly in the case of atomic modal interpretations, must break Lorentz-invariance [23]. Most of Dieks' recent Letter [1] is concerned to address this dynamics problem by appropriating ideas from the decoherent histories approach to quantum theory (cf. [24]). However, we shall focus here entirely on the viability of Dieks' new proposal for picking out definite-valued observables of a relativistic quantum field that are asso-

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ciated with approximately point-sized regions of Minkowski spacetime ([1], Sec. 5).

2 Critique of Dieks' Proposal

Dieks' stated aim is to see if the modal interpretation can achieve sensible results in the context of quantum field theory. For this purpose, he adopts the formalism of algebraic quantum field theory because of its generality [25]–[27]. In the concrete 'Haag-Araki' approach, one supposes that a quantum field on Minkowski spacetime M will associate to each bounded open region $O \subseteq M$ a von Neumann algebra $\mathcal{R}(O)$ of observables measurable in that region, where the collection $\{\mathcal{R}(O) : O \subseteq M\}$ acts irreducibly on some fixed Hilbert space H. It is then natural to treat each open region O and associated algebra $\mathcal{R}(O)$ as a quantum system in its own right. Given any (normal) state ρ of the field (where ρ is a state functional on $\mathcal{B}(\mathsf{H})$), we can then ask which observables in $\mathcal{R}(O)$ are picked out as definite-valued by the restriction, ρ_O, of the state ρ to $\mathcal{R}(O)$.

The difficulty for the modal interpretation (though Dieks himself does not put it this way) is that when O has nonempty spacelike complement O', $\mathcal{R}(O)$ will typically be a type III factor that contains no nonzero finite projections ([25], Sec. V.6; [27], Sec. 17.2). Because of this, $\mathcal{R}(O)$ cannot contain compact operators, like density operators, all of whose (non-null) spectral projections are finite-dimensional. As a result, there is no density operator in $\mathcal{R}(O)$ that can represent ρ_O. Moreover, if we try to apply the standard modal prescription based on Eq. (1) to a density operator in $\mathcal{B}(\mathsf{H})$ that agrees with ρ_O, there is no guarantee that the resulting set of observables will pick out a subalgebra of $\mathcal{R}(O)$, and we will be left with nothing to say about which observables have definite values in O. The moral Dieks draws from this is that "We can therefore not take the open spacetime regions and their algebras as fundamental, if we want an interpretation in terms of (more or less) localized systems whose properties would specify an event" ([1], p. 322). We shall see in the next section that this conclusion is overly pessimistic.

In any case, Dieks' strategy for dealing with the problem is to exploit the fact that, in most models of the axioms of algebraic quantum field theory, the local algebras associated with diamond shaped spacetime regions (i.e., regions given by interior of the intersection of the causal future and past of two space-

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time points) have the split property. The property is that for any two concentric diamond shaped regions $\Diamond_r$, $\Diamond_{r+\epsilon} \subseteq M$, with radii r and $r + \epsilon$, there is a type I 'interpolating' factor $\mathcal{N}_{r+\epsilon}$ such that $\mathcal{R}(\Diamond_r) \subset \mathcal{N}_{r+\epsilon} \subset \mathcal{R}(\Diamond_{r+\epsilon})$. Now since $\mathcal{N}_{r+\epsilon} \approx \mathcal{B}(\overline{\mathsf{H}})$ for some Hilbert space $\overline{\mathsf{H}}$, we know there is always a unique density operator $D_{r+\epsilon} \in \mathcal{B}(\overline{\mathsf{H}})$ that agrees with ρ on $\mathcal{N}_{r+\epsilon}$, and therefore with $\rho_{\Diamond_r}$. The proposal is, then, to take both r and ϵ to be fixed small numbers and apply the prescription in Eq. (1) to this density operator $D_{r+\epsilon}$, yielding a definite-valued subalgebra $\mathcal{M}_{r+\epsilon} \subseteq \mathcal{N}_{r+\epsilon}$. This, according to Dieks, should give an approximation indication of which observables have definite values at the common origin of the two diamonds in "the classical limiting situation in which classical field and particle concepts become approximately applicable" ([1], p. 323). Thus, Dieks proposes to build up an atomic modal interpretation of the field as follows. (i) We subdivide (approximately) the whole of spacetime M into a collection of non-overlapping diamond regions $\Diamond_r$, with some fixed small radius r. (ii) We choose some fixed small ϵ and an interpolating factor $\mathcal{N}_{r+\epsilon}$ for each diamond, using its ρ-induced density operator $D_{r+\epsilon}$ and Eq. (1) to determine the definite-valued observables $\mathcal{M}_{r+\epsilon} \subset \mathcal{R}(\Diamond_{r+\epsilon})$ to be loosely associated with the origin. (iii) Finally, we build up definite-valued observables associated with collections of diamonds, and define their joint probabilities in the usual way via Born's rule (defining transition probabilities between values of observables associated with timelike-separated diamonds using the familiar multi-time generalization of that rule employed in the decoherent histories approach).

As things stand, there is much arbitrariness in this proposal that enters into the stages (i) and (ii). Dieks himself recognizes the arbitrariness in the size of the partition of M chosen. He also acknowledges that this arbitrariness cannot be eliminated by passing to the limit $r, \epsilon \to 0$, because the intersection of the algebras associated with any collection of concentric diamonds is always the trivial algebra $\mathfrak{C}I$ [28]. Indeed, one would have thought that this undermines any attempt to formulate an *atomic* modal interpretation in this context, because it forces the choice of 'atomic diamonds' in the partition of M to be essentially arbitrary. Dieks appears to suggest that this arbitrariness will become unimportant in some classical limit of relativistic quantum field theory in which we should recover "the classical picture according to which field values are attached to spacetime points" ([1], p. 325). But it is not sufficient for the success of a proposal for interpreting a relativistic *quantum* field theory that the interpretation give sensible results in the limit of *classical* relativistic (or nonrelativistic) field theory. Indeed, the only relevant

limit would appear to be the nonrelativistic limit; i.e., Galilean quantum field theory. But, there, one still needs to spatially smear "operator-valued" fields at each point to obtain a well-defined algebra of observables in a spatial region [29], so there will again be no natural choice to make for atomic spatial regions or algebras.

There is also another, more troubling, degree of arbitrariness at step (ii) in the choice of the type I interpolating factor $\mathcal{N}_{r+\epsilon}$ about each origin point. For any fixed partition and fixed $r, \epsilon > 0$, we can always sub-divide the interval $(r, r + \epsilon)$ further, and then the split property implies the existence of a pair of interpolating type I factors satisfying

$$\mathcal{R}(\Diamond_r) \subset \mathcal{N}_{r+\epsilon/2} \subset \mathcal{R}(\Diamond_{r+\epsilon/2}) \subset \mathcal{N}_{r+\epsilon} \subset \mathcal{R}(\Diamond_{r+\epsilon}). \tag{2}$$

The problem is that we now face a nontrivial choice deciding which of these factors' ρ-induced density operators to use to pick out the definite-valued observables in the state ρ associated with the origin. If we pick $D_{r+\epsilon} \in \mathcal{N}_{r+\epsilon}$, then by (1) all observables $A \in \mathcal{N}_{r+\epsilon}$ that share the same spectral projections as $D_{r+\epsilon}$ will have definite values. However, *no* such A can lie in $\mathcal{M}_{r+\epsilon/2} \subsetneq \mathcal{N}_{r+\epsilon/2}$, nor even in $\mathcal{R}(\Diamond_{r+\epsilon/2})$. The reason is that A's spectral projections are *finite in* $\mathcal{N}_{r+\epsilon}$. So if those projections were also in the type III algebra $\mathcal{R}(\Diamond_{r+\epsilon/2})$, they would have to be infinite in $\mathcal{R}(\Diamond_{r+\epsilon/2})$, and therefore also infinite projections in $\mathcal{N}_{r+\epsilon}$ — which is impossible.

Clearly we can sub-divide the interval $(r, r + \epsilon)$ arbitrarily many times in this way and obtain a monotonically decreasing sequence of type I factors satisfying $\mathcal{R}(\Diamond_{r+\epsilon/2^{n+1}}) \subset \mathcal{N}_{r+\epsilon/2^n} \subset \mathcal{R}(\Diamond_{r+\epsilon/2^n})$ that all interpolate between $\mathcal{R}(\Diamond_r)$ and $\mathcal{R}(\Diamond_{r+\epsilon})$. The sequence $\{\mathcal{N}_{r+\epsilon/2^n}\}_{n=0}^{\infty}$ has no least member, and its greatest member, $\mathcal{N}_{r+\epsilon}$, is arbitrary, because we could also further sub-divide the interval $(r+\epsilon/2, r+\epsilon)$ ad infinitum. Thus there is no natural choice of interpolating factor for picking out the observables definite at the origin, even if we restrict ourselves to a 'nice' decreasing sequence of interpolating factors of the form $\{\mathcal{N}_{r+\epsilon/2^n}\}_{n=0}^{\infty}$.

On the other hand, it can actually be shown that $\mathcal{R}(\Diamond_r) = \bigcap_{n=0}^{\infty} \mathcal{N}_{r+\epsilon/2^n}$ ([26], pp. 12-3; [27], p. 426). Furthermore, suppose $\{\mathcal{N}_{r+\epsilon_n}\}_{n=0}^{\infty}$ is *any other* decreasing type I sequence satisfying

$$\mathcal{R}(\Diamond_{r+\epsilon_{n+1}}) \subset \mathcal{N}_{r+\epsilon_n} \subset \mathcal{R}(\Diamond_{r+\epsilon_n}), \ \epsilon_0 = \epsilon, \ \epsilon_n > \epsilon_{n+1}, \ \lim \epsilon_n = 0. \tag{3}$$

Then, since for any n there will be a sufficiently large n' such that $\mathcal{N}_{r+\epsilon_{n'}} \subset \mathcal{N}_{r+\epsilon/2^n}$ (and vice-versa), clearly $\mathcal{R}(\Diamond_r) = \bigcap_{n=0}^{\infty} \mathcal{N}_{r+\epsilon_n}$, and this intersection

will also be independent of ϵ. It would seem, then, that the natural way to avoid choosing between the myriad type I factors that interpolate between $\mathcal{R}(\diamondsuit_r)$ and $\mathcal{R}(\diamondsuit_{r+\epsilon})$ is to take the observables definite-valued at the origin to be those in the intersection $\bigcap_{n=0}^{\infty} \mathcal{M}_{r+\epsilon_n} \subseteq \mathcal{R}(\diamondsuit_r)$ (where, as before, $\mathcal{M}_{r+\epsilon_n}$ is the modal subalgebra of $\mathcal{N}_{r+\epsilon_n}$ determined via (1) by the density operator in $\mathcal{N}_{r+\epsilon_n}$ that represents ρ). Indeed, Dieks' suggestion appears to be that when we take successively smaller values for ϵ (holding r fixed), and choose a type I interpolating factor at each stage, we should be getting progressively better approximations to the set of observables that are truly definite at the origin. What better candidate for that set can there be than an intersection like $\bigcap_{n=0}^{\infty} \mathcal{M}_{r+\epsilon_n}$?

Unfortunately, we have no guarantee that *this* intersection, unlike $\bigcap_{n=0}^{\infty} \mathcal{N}_{r+\epsilon_n}$ itself, is independent of the particular sequence $\{\epsilon_n\}$ or its starting value $\epsilon_0 = \epsilon$. The reason any intersection of form $\bigcap_{n=0}^{\infty} \mathcal{N}_{r+\epsilon_n}$ is so independent is because $\mathcal{N}_{r+\epsilon_n} \supset \mathcal{N}_{r+\epsilon_{n+1}}$ for all n. But this does *not* imply $\mathcal{M}_{r+\epsilon_n} \supset \mathcal{M}_{r+\epsilon_{n+1}}$. To take just a trivial example: when ρ is a pure state of $\mathcal{N}_{r+\epsilon_n}$ that induces a mixed state on the proper subalgebra $\mathcal{N}_{r+\epsilon_{n+1}}$ of $\mathcal{N}_{r+\epsilon_n}$, $\mathcal{M}_{r+\epsilon_{n+1}}$ will contain observables with dispersion in the state ρ, but $\mathcal{M}_{r+\epsilon_n}$ will not.

We conclude that there is little prospect of eliminating the arbitrariness in Dieks' proposal and making it well-defined. One ought to look for another, *intrinsic* way to pick out the definite-valued observables in $\mathcal{R}(\diamondsuit_r)$ that does not depend on special assumptions such as the split property.

3 The Modal Interpretation for Arbitrary von Neumann Algebras

There are two salient features of the algebra $\mathcal{M}_S$ in Eq. (1) that make it an attractive set of definite-valued observables to modal interpreters. First, $\mathcal{M}_S$ is locally determined by the quantum state D_S of system S together with the structure of its algebra of observables. In particular, there is no need to add any additional structure to the standard formalism of quantum theory to pick out S's properties. Second, the restriction of the state D_S to the subalgebra $\mathcal{M}_S$ is a mixture of dispersion-free states (given by the density operators one obtains by renormalizing the (non-null) spectral projections of D_S). This second feature is what makes it possible to think of the observables in $\mathcal{M}_S$ as possessing definite values distributed in accordance with standard

Born rule statistics [9]. Let us see, then, whether we can generalize these two features to come up with a proposal for the definite-valued observables of a system described by an arbitrary von Neumann algebra $\mathcal{R}$ (acting on some Hilbert space H) in an arbitrary state ρ of $\mathcal{R}$.

Generally, a state ρ of $\mathcal{R}$ will be a mixture of dispersion-free states on a subalgebra $\mathcal{S} \subseteq \mathcal{R}$ just in case there is a probability measure μ_ρ on the space Λ of dispersion-free states of $\mathcal{S}$ such that

$$\rho(A) = \int_\Lambda \omega_\lambda(A)d\mu_\rho(\lambda), \text{ for all } A \in \mathcal{S}, \qquad (4)$$

where $\omega_\lambda(A^2) = \omega_\lambda(A)^2$ for all self-adjoint elements $A \in \mathcal{S}$. This somewhat cumbersome condition turns out to be equivalent ([10], Prop. 2.2(ii)) to simply requiring that

$$\rho([A, B]^*[A, B]) = 0 \text{ for all } A, B \in \mathcal{S}. \qquad (5)$$

In particular, ρ can always be represented as a mixture of dispersion-free states on any abelian subalgebra $\mathcal{S} \subseteq \mathcal{R}$. Conversely, if ρ is a faithful state of $\mathcal{R}$, i.e., ρ maps no nonzero positive elements of $\mathcal{R}$ to zero, then the *only* subalgebras that allow ρ to be represented as a mixture of dispersion-free states are the abelian ones. There is now an easy way to pick out a subalgebra $\mathcal{S} \subseteq \mathcal{R}$ with this property, using only ρ and the algebraic operations available within $\mathcal{R}$.

Consider the following two mathematical objects explicitly defined in terms of $\mathcal{R}$ and ρ. First, the support projection of the state ρ in $\mathcal{R}$, defined by

$$P_{\rho,\mathcal{R}} \equiv \wedge\{P = P^2 = P^* \in \mathcal{R} : \rho(P) = 1\}, \qquad (6)$$

which is simply the smallest projection in $\mathcal{R}$ that the state ρ 'makes true'. Second, there is the centralizer subalgebra of the state ρ in $\mathcal{R}$, defined by

$$\mathcal{C}_{\rho,\mathcal{R}} \equiv \{A \in \mathcal{R} : \rho([A, B]) = 0 \text{ for all } B \in \mathcal{R}\}. \qquad (7)$$

For any von Neumann algebra $\mathcal{K}$, let $\mathcal{Z}(\mathcal{K}) \equiv \mathcal{K} \cap \mathcal{K}'$, the center algebra of $\mathcal{K}$. Then it is reasonable for the modal interpreter to take as definite-valued all the observables that lie in the direct sum

$$\mathcal{S} = \mathcal{M}_{\rho,\mathcal{R}} \equiv P_{\rho,\mathcal{R}}^\perp \mathcal{R} P_{\rho,\mathcal{R}}^\perp + \mathcal{Z}(\mathcal{C}_{\rho,\mathcal{R}})P_{\rho,\mathcal{R}} \subseteq \mathcal{R}, \qquad (8)$$

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where the algebra in the first summand acts on the subspace $P_{\rho,\mathcal{R}}^{\perp}\mathsf{H}$ and that of the second acts on $P_{\rho,\mathcal{R}}\mathsf{H}$. The state ρ is a mixture of dispersion-free states on $\mathcal{M}_{\rho,\mathcal{R}}$, by (5), because ρ maps all elements of the form $P_{\rho,\mathcal{R}}^{\perp}\mathcal{R}P_{\rho,\mathcal{R}}^{\perp}$ to zero, and the product of the commutators of any two elements of $\mathcal{Z}(\mathcal{C}_{\rho,\mathcal{R}})P_{\rho,\mathcal{R}}$ also gets mapped to zero, for the trivial reason that $\mathcal{Z}(\mathcal{C}_{\rho,\mathcal{R}})$ is abelian.

The set $\mathcal{M}_{\rho,\mathcal{R}}$ directly generalizes the algebra of Eq. (1) to the non-type I case where the algebra of observables of the system does not contain a density operator representative of the state ρ. Assuming the type I case, $\mathcal{R}$ $\approx \mathcal{B}(\overline{\mathsf{H}})$ for some Hilbert space $\overline{\mathsf{H}}$, ρ is given by a density operator D on $\overline{\mathsf{H}}$, $P_{\rho,\mathcal{R}}$ is equivalent to the range projection of D, and $\mathcal{Z}(\mathcal{C}_{\rho,\mathcal{R}}) \approx \mathcal{Z}(\mathcal{C}_{D,\mathcal{B}(\overline{\mathsf{H}})})$. So to show that $\mathcal{M}_{\rho,\mathcal{R}}$ is isomorphic to the algebra of Eq. (1), it suffices to establish that $\mathcal{Z}(\mathcal{C}_{D,\mathcal{B}(\overline{\mathsf{H}})}) = D''$. It is easy to see that $\mathcal{C}_{D,\mathcal{B}(\overline{\mathsf{H}})} = D'$ (invoking cyclicity and positive-definiteness of the trace), thus $\mathcal{Z}(\mathcal{C}_{D,\mathcal{B}(\overline{\mathsf{H}})}) = D' \cap D''$. However, since D' always contains a maximal abelian subalgebra of $\mathcal{B}(\overline{\mathsf{H}})$ (viz., that generated by the projections onto any complete orthonormal basis of eigenvectors for D), we always have $D'' \subseteq D'$.

Choosing $\mathcal{M}_{\rho,\mathcal{R}}$ is certainly not the only way to pick a subalgebra $\mathcal{S} \subseteq \mathcal{R}$ that is definable in terms of ρ and $\mathcal{R}$ and allows ρ to be represented as a mixture of dispersion-free states. There is the obvious orthodox alternative one can always consider, viz., the definite algebra of ρ in $\mathcal{R}$,

$$\mathcal{S} = \mathcal{O}_{\rho,\mathcal{R}} \equiv \{A \in \mathcal{R} : \rho(AB) = \rho(A)\rho(B) \text{ for all } B \in \mathcal{R}\}, \qquad (9)$$

which coincides with the complex span of all self-adjoint members of $\mathcal{R}$ on which ρ is dispersion-free ([10], p. 2445). Note, however, that we always have $\mathcal{O}_{\rho,\mathcal{R}} \subseteq \mathcal{M}_{\rho,\mathcal{R}}$. Indeed the problem is that the orthodox choice $\mathcal{O}_{\rho,\mathcal{R}}$ generally will contain far too few definite-valued observables to solve the measurement problem. For example, when ρ is faithful — and there will always be a norm dense set of states of $\mathcal{R}$ that are — we get just $\mathcal{O}_{\rho,\mathcal{R}} = \mathfrak{C}I$. Thus it is natural for a modal interpreter to require that the choice of $\mathcal{S} \subseteq \mathcal{R}$ be maximal. In the case where ρ is faithful, we now show that this singles out the choice $\mathcal{S} = \mathcal{M}_{\rho,\mathcal{R}} = \mathcal{Z}(\mathcal{C}_{\rho,\mathcal{R}})$ uniquely (and we conjecture that a similar uniqueness result holds for the more general expression for $\mathcal{M}_{\rho,\mathcal{R}}$ in Eq. (8), using the fact that an arbitrary state ρ always renormalizes to a faithful state on $P_{\rho,\mathcal{R}}\mathcal{R}P_{\rho,\mathcal{R}}$).

Proposition 1 *Let $\mathcal{R}$ be a von Neumann algebra and ρ a faithful normal state of $\mathcal{R}$ with centralizer $\mathcal{C}_{\rho,\mathcal{R}} \subseteq \mathcal{R}$. Then $\mathcal{Z}(\mathcal{C}_{\rho,\mathcal{R}})$, the center of $\mathcal{C}_{\rho,\mathcal{R}}$, is the unique subalgebra $\mathcal{S} \subseteq \mathcal{R}$ such that:*

1. *The restriction of ρ to $\mathcal{S}$ is a mixture of dispersion-free states.*

2. $\mathcal{S}$ *is definable solely in terms of ρ and the algebraic structure of $\mathcal{R}$.*

3. $\mathcal{S}$ *is maximal with respect to properties 1. and 2.*

Proof: By 3., it suffices to show than any $\mathcal{S} \subseteq \mathcal{R}$ satisfying 1. and 2. is contained in $\mathcal{Z}(\mathcal{C}_{\rho,\mathcal{R}})$. And for this, it suffices (because von Neumann algebras are generated by their projections) to show that $\underline{\mathcal{S}}$, the subset of projections in $\mathcal{S}$, is contained in $\mathcal{Z}(\mathcal{C}_{\rho,\mathcal{R}})$. Recall also that, as a consequence of 1. and the faithfulness of ρ, $\mathcal{S}$ must be abelian. And in virtue of 2., any automorphism $\sigma : \mathcal{R} \longrightarrow \mathcal{R}$ that preserves the state ρ in the sense that $\rho \circ \sigma = \rho$, must leave the set $\mathcal{S}$ (not necessarily pointwise) invariant, i.e., $\sigma(\mathcal{S}) = \mathcal{S}$.

$\underline{\mathcal{S}} \subseteq \mathcal{C}'_{\rho,\mathcal{R}}$. Any unitary operator $U \in \mathcal{C}_{\rho,\mathcal{R}}$ defines an inner automorphism on $\mathcal{R}$ that leaves ρ invariant, therefore $U\mathcal{S}U^{-1} = \mathcal{S}$. Since $\mathcal{S}$ is abelian, $[UPU^{-1}, P] = 0$ for each $P \in \underline{\mathcal{S}}$ and all unitary $U \in \mathcal{C}_{\rho,\mathcal{R}}$. By Lemma 4.2 of [10] (with $\mathfrak{V} = \mathcal{C}''_{\rho,\mathcal{R}} = \mathcal{C}_{\rho,\mathcal{R}}$), this implies that $P \in \mathcal{C}'_{\rho,\mathcal{R}}$.

$\underline{\mathcal{S}} \subseteq \mathcal{C}_{\rho,\mathcal{R}}$. Since ρ is faithful, there is a one-parameter group $\{\sigma_t : t \in \mathfrak{R}\}$ of automorphisms of $\mathcal{R}$ — the modular automorphism group of $\mathcal{R}$ determined by ρ ([30], Sec. 9.2) — leaving ρ invariant. Since $\mathcal{C}_{\rho,\mathcal{R}}$ consists precisely of the fixed points of the modular group ([30], Prop. 9.2.14), it suffices to show that it leaves the individual elements of $\underline{\mathcal{S}}$ fixed. For this, we use the fact that the modular group satisfies the KMS condition with respect to ρ: for each $A, B \in \mathcal{R}$, there is a complex-valued function f, bounded and continuous on the strip $\{z \in \mathfrak{C} : 0 \leq \operatorname{Im} z \leq 1\}$ in the complex plane, and analytic on the interior of that strip, such that

$$f(t) = \rho(\sigma_t(A)B), \ f(t+i) = \rho(B\sigma_t(A)), \ t \in \mathfrak{R}. \tag{10}$$

In fact, we shall need only one simple consequence of the KMS condition, viz., if $f(t) = f(t+i)$ for all $t \in \mathfrak{R}$, then f is constant ([30], p. 611).

Fix an arbitrary projection $P \in \underline{\mathcal{S}}$. Since the modular automorphism group must leave $\mathcal{S}$ as a whole invariant, and $\mathcal{S}$ is abelian, $[\sigma_t(P), P^\perp] = 0$ for all $t \in \mathfrak{R}$. However, there exists a function f with the above properties such that

$$f(t) = \rho(\sigma_t(P)P^\perp) = \rho(P^\perp \sigma_t(P)) = f(t+i), \ t \in \mathfrak{R},$$

so it follows that f is constant. In particular, since $\sigma_0(P)P^\perp = PP^\perp = 0$, f is identically zero, and $\rho(\sigma_t(P)P^\perp) = 0$ for all $t \in \mathfrak{R}$. And since $\sigma_t(P)$ and

$P^\perp$ are commuting projections, their product is a (positive) projection, so that the faithfulness of ρ requires that $\sigma_t(P)P^\perp = 0$, or equivalently $\sigma_t(P) = \sigma_t(P)P$, for all $t \in \mathfrak{R}$. Running through the exact same argument, starting with $P^\perp \in \underline{\mathcal{S}}$ in place of P, yields $\sigma_t(P^\perp) = \sigma_t(P^\perp)P^\perp$, or equivalently, $\sigma_t(P)P = P$, for all $t \in \mathfrak{R}$. Together with $\sigma_t(P) = \sigma_t(P)P$, this implies that $\sigma_t(P) = P$ for all $t \in \mathfrak{R}$. QED.

The choice $\mathcal{M}_{\rho,\mathcal{R}} = \mathcal{Z}(\mathcal{C}_{\rho,\mathcal{R}})$ has another feature that generalizes a natural consequence of the modal interpretation of nonrelativistic quantum theory. Suppose the universal state $x \in \mathsf{H}_U$ defines a faithful state ρ_x on both $\mathcal{B}(\mathsf{H}_S)$ and $\mathcal{B}(\mathsf{H}_{\overline{S}})$. This requires that $\dim \mathsf{H}_S = \dim \mathsf{H}_S = n$ (possibly ∞), and, furthermore, that any Schmidt decomposition of the state vector x relative to the factorization $\mathsf{H}_U = \mathsf{H}_S \otimes \mathsf{H}_{\overline{S}}$ takes the form

$$x = \sum_{i=1}^{n} c_i v_i \otimes w_i, \ c_i \neq 0 \text{ for all } i = 1 \text{ to } n, \tag{11}$$

where the vectors v_i and w_i are complete orthonormal bases in their respective spaces. As is well-known, for each distinct eigenvalue $\widetilde{\lambda}_j$ for D_S, the span of the vectors v_i for which $|c_i|^2 = \widetilde{\lambda}_j$ coincides with the range of the $\widetilde{\lambda}_j$-eigenprojection of D_S, and similarly for $D_{\overline{S}}$. Consequently, there is a natural bijective correspondence between the properties represented by the projections in the two sets $\mathcal{M}_S = D_S''$ and $\mathcal{M}_{\overline{S}} = D_{\overline{S}}''$: any definite property S happens to possess is strictly correlated to a unique property of its environment $\overline{S}$ that occurs with the same frequency. More formally, for any $P \in \mathcal{M}_S$, there is a unique $\overline{P} \in \mathcal{M}_{\overline{S}}$ satisfying

$$(x, P\overline{P}x) = (x, Px) = (x, \overline{P}x). \tag{12}$$

To see this, note that any $P \in \mathcal{M}_S$ (in this case, the set of all functions of D_S) is a sum of spectral projections of D_S. Let $\overline{P} \in \mathcal{M}_{\overline{S}}$ be the sum of the corresponding spectral projections of $D_{\overline{S}}$ for the same eigenvalues. Then it is evident from the form of the state expansion in (11) that $\overline{P}$ has the property in (12), and no other projection in $\mathcal{M}_{\overline{S}}$ does. This has led some non-atomic modal interpreters, such as Kochen [11], to interpret each property P of S, not as a property that S possesses absolutely, but only in relation to its environment $\overline{S}$ possessing the corresponding property $\overline{P}$.

For a general von Neumann factor $\mathcal{R}$, $(\mathcal{R} \cup \mathcal{R}')'' = \mathcal{B}(\mathsf{H})$ need not be isomorphic to the tensor product $\mathcal{R} \otimes \mathcal{R}'$ (particularly when $\mathcal{R}$ is type III,

for then $\mathcal{R} \otimes \mathcal{R}'$ must be type III as well). Therefore, there is no direct analogue of a Schmidt decomposition for a pure state $x \in \mathsf{H}$ relative to the factorization $(\mathcal{R} \cup \mathcal{R}')''$ of $\mathcal{B}(\mathsf{H})$. Nevertheless, we show next that there is still the same strict correlation between definite properties in $\mathcal{M}_{\rho_x,\mathcal{R}} = \mathcal{Z}(\mathcal{C}_{\rho_x,\mathcal{R}})$ and $\mathcal{M}_{\rho_x,\mathcal{R}'} = \mathcal{Z}(\mathcal{C}_{\rho_x,\mathcal{R}'})$.

Proposition 2 *Let $\mathcal{R}$ be a von Neumann algebra acting on a Hilbert space H, and suppose $x \in \mathsf{H}$ induces a state ρ_x that is faithful on both $\mathcal{R}$ and $\mathcal{R}'$. Then for any projection $P \in \mathcal{Z}(\mathcal{C}_{\rho_x,\mathcal{R}})$, there is a unique projection $\overline{P} \in \mathcal{Z}(\mathcal{C}_{\rho_x,\mathcal{R}'})$ such that $(x, P\overline{P}x) = (x, Px) = (x, \overline{P}x)$.*

Proof: For any fixed $A \in \mathcal{R}$, call an element $B \in \mathcal{R}'$ a double for A (in state x) just in case $Ax = Bx$ and $A^*x = B^*x$. By an elementary application of modular theory, Werner ([31], Sec. II) has shown that $\mathcal{C}_{\rho_x,\mathcal{R}}$ consists precisely of those elements of $\mathcal{R}$ with doubles in $\mathcal{R}'$ (with respect to x). Moreover, the double of any element of $\mathcal{R}$ clearly has to be unique, by the faithfulness of ρ_x on $\mathcal{R}'$. Now it is easy to see (again using the faithfulness of ρ_x) that the double of any projection $P \in \mathcal{C}_{\rho_x,\mathcal{R}}$ is a projection $\overline{P} \in \mathcal{C}_{\rho_x,\mathcal{R}'}$ satisfying (12). We claim that whenever $P \in \mathcal{C}'_{x,\mathcal{R}}$, we have $\overline{P} \in \mathcal{C}'_{x,\mathcal{R}'}$. For this, it suffices to show $P \in \mathcal{C}'_{x,\mathcal{R}}$ implies that for arbitrary $B \in \mathcal{C}_{x,\mathcal{R}'}$, $[\overline{P}, B]x = 0$ (and then $[\overline{P}, B]$ itself is zero, since ρ_x is faithful). Letting $A \in \mathcal{C}_{x,\mathcal{R}}$ be the double of B in $\mathcal{R}$, we get

$$\overline{P}Bx - B\overline{P}x = \overline{P}Ax - BPx = A\overline{P}x - PBx = APx - PAx = 0, \quad (13)$$

as required. Finally, were there another projection $\widetilde{P} \in \mathcal{Z}(\mathcal{C}_{\rho_x,\mathcal{R}'})$ satisfying (12), then by exploiting the fact that $\overline{P}$ is P's double in $\mathcal{R}'$, we get $(x, \overline{P}\widetilde{P}x) = (x, \overline{P}x) = (x, \widetilde{P}x)$; or, equivalently,

$$(x, \overline{P}\widetilde{P}^{\perp}x) = (x, \overline{P}^{\perp}\widetilde{P}x) = 0. \quad (14)$$

Since $\mathcal{Z}(\mathcal{C}_{\rho_x,\mathcal{R}'})$ is abelian, both $\overline{P}\widetilde{P}^{\perp}$ and $\overline{P}^{\perp}\widetilde{P}$ are (positive) projections in $\mathcal{R}$. But as ρ_x is faithful on $\mathcal{R}$, Eqs. (14) entail that $\overline{P}\widetilde{P}^{\perp} = \overline{P}^{\perp}\widetilde{P} = 0$, which in turn implies that $\overline{P} = \widetilde{P}$, as required for uniqueness. *QED*.

Let us return now to the problem of picking out a set of definite-valued observables localized in a diamond region with associated algebra $\mathcal{R}(\Diamond_r)$. Let ρ be any pure state of the field that induces a faithful state on $\mathcal{R}(\Diamond_r)$; for example, ρ could be the vacuum or any one of the dense set of states

of a field with bounded energy (by the Reeh-Schlieder theorem — see [26], Thm. 1.3.1). By Proposition 1, the definite-valued observables in $\mathcal{R}(\diamondsuit_r)$ are simply those in the subalgebra $\mathcal{Z}(\mathcal{C}_{\rho,\mathcal{R}(\diamondsuit_r)})$. Note that this proposal yields observables all of which have an exact spacetime localization within the open set $\diamondsuit_r$ and are picked out intrinsically by the local algebra $\mathcal{R}(\diamondsuit_r)$ and the field state ρ. Contrary to Dieks' pessimistic conclusion, we *can* take open spacetime regions as fundamental for determining the definite-valued observables. In fact, this proposal works independent of the size of r, and so could also be embraced by *non*-atomic modal interpreters not wishing to commit themselves to a particular partition of the field into subsystems (or to thinking from the outset in terms of approximately point-localized field observables). Finally, note that since the algebra of a diamond region $\diamondsuit_r$ satisfies duality with respect to the algebra of its spacelike complement $\diamondsuit_r'$, i.e., $\mathcal{R}(\diamondsuit_r)' = \mathcal{R}(\diamondsuit_r')$ ([25], p. 145), Proposition 2 tells us that there is a natural bijective correspondence between the properties in $\mathcal{Z}(\mathcal{C}_{\rho,\mathcal{R}(\diamondsuit_r)})$ and strictly correlated properties in $\mathcal{Z}(\mathcal{C}_{\rho,\mathcal{R}(\diamondsuit_r')})$ associated with the complement region.

4   A Potential Difficulty with Ergodic States

We have seen that there *is*, after all, a well-motivated and unambiguous prescription extending the standard modal interpretation of nonrelativistic quantum theory to the local algebras of quantum field theory. We also, now, have a natural standard of comparison with Galilean quantum field theory. At least in the case of free fields, it is possible to build up local algebras in M from spatially smeared "field algebras" defined on spacelike hyperplanes in M. A diamond region corresponds to the domain of dependence of a spatial region in a hyperplane, and it can be shown that the algebra of that spatial region will also be type III and, indeed, *coincide* with its domain of dependence algebra ([26], Prop. 3.3.2, Thm. 3.3.4). These type III spatial algebras in M, and the definite-valued observables therein, are what should be compared, in the nonrelativistic limit, to the corresponding equal time spatial algebras defined on simultaneity slices of Galilean spacetime. Unfortunately, since the algebras in the Galilean case are invariably type I ([26], p. 35), this limit is bound to be mathematically singular, and its *physical* characterization needs to be dealt with carefully. But this is a problem for *any* would-be interpreter of relativistic quantum field theory, not just modal

interpreters. All we should require of them, at this stage, is that they be able to say something sensible in the relativistic case about the local observables with definite values (which was, indeed, Dieks' original goal). However, as we now explain, it is not clear whether even this goal can be attained.

If $\mathcal{R} \approx \mathcal{B}(\overline{\mathsf{H}})$ is type I, it possesses at most one faithful state ρ such that $\mathcal{Z}(\mathcal{C}_{\rho,\mathcal{R}}) = \mathfrak{C}I$. This is easy to see, because if $D \in \mathcal{B}(\overline{\mathsf{H}})$ represents ρ, $\mathcal{Z}(\mathcal{C}_{\rho,\mathcal{R}}) \approx D''$, and $D'' = \mathfrak{C}I$ implies that D itself must be a multiple of the identity. So when $\overline{\mathsf{H}}$ is finite-dimensional, we must have $D = I/\dim\overline{\mathsf{H}}$, the unique maximally mixed state, and in the infinite-dimensional case, no such density operator even exists. Elsewhere Dieks [32] has argued convincingly that there is no problem when a system, occupying a maximally mixed state, possesses only trivial properties, because such states are rare and highly unstable under environmental decoherence (cf. [2], pp. 99-100). However, the situation is quite different for the local algebras of algebraic quantum field theory.

In all physically reasonable models of the axioms of the theory, every local algebra $\mathcal{R}(O)$ is isomorphic to the unique (up to isomorphism) hyperfinite type III_1 factor ([25], Sec. V.6; [27], Sec. 17.2). In that case, there is a novel way to obtain $\mathcal{Z}(\mathcal{C}_{\rho,\mathcal{R}(O)}) = \mathfrak{C}I$, namely, when the state ρ of $\mathcal{R}(O)$ is an ergodic state [33, 34], i.e., ρ possesses a trivial centralizer in $\mathcal{R}(O)$. (Were $\mathcal{R}$ a nonabelian type I factor, this would be impossible, since $D' = \mathfrak{C}I$ implies $D'' = \mathcal{B}(\overline{\mathsf{H}}) \approx \mathcal{R}$, which is patently false.) In fact, we have the following result.

Proposition 3 *If $\mathcal{R}$ is the hyperfinite type III_1 factor, there is a norm dense set of unit vectors in the Hilbert space H on which $\mathcal{R}$ acts that induce faithful states on $\mathcal{R}$ with trivial centralizers (i.e., ergodic states).*

Proof: First recall the following facts provable from the axioms of algebraic quantum field theory: (i) the vacuum state of a field on M has a trivial centralizer in the algebra of any Rindler wedge ([27], Sec. 16.1.1); (ii) the vacuum is faithful for any wedge algebra (by the Reeh-Schlieder theorem); and (iii) wedge algebras are hyperfinite type III_1 factors ([27], Ex. 16.2.14, pp. 426-7). Since being faithful and having a trivial centralizer are isomorphic invariants, it follows that *any* instantiation $\mathcal{R}$ of the hyperfinite type III_1 factor possesses at least one faithful normal state ρ with trivial centralizer (even when $\mathcal{R}$ is the algebra of a *bounded* open region, like a diamond). Now since $\mathcal{R}$ is type III, all its states are vector states (combine [35], Cor. 2.9.28 with [30], Thm. 7.2.3); in particular, $\rho = \rho_x$ for some unit vector $x \in \mathsf{H}$.

Furthermore, by the homogeneity of the state space of type III$_1$ factors ([36], Cor. 6), the set of all unit vectors of the form $UU'x$, with $U \in \mathcal{R}$ and $U' \in \mathcal{R}'$ unitary operators, lies dense in H. But clearly any such vector must again induce a faithful state on $\mathcal{R}$ with trivial centralizer. QED.

Combining Propositions 1 and 3, there will be a whole host of states of any relativistic quantum field in which the modal interpreter is forced to assert that *no* nontrivial local observables have definite values! Note, however, that while the set of field states ergodic for any given type III$_1$ local algebra is always dense, this does not automatically imply that such states are typical or generic. Indeed, results of Summers and Werner ([37], particularly Cor. 2.4) imply that for any local diamond algebra $\mathcal{R}(\Diamond ß_r)$, there will *also* always be a dense set field states whose centralizers in $\mathcal{R}(\Diamond_r)$ contain the hyperfinite type II$_1$ factor, and so will *not* be trivial. Still, the modal interpreter needs to provide some physical reason for neglecting the densely many field states that *do* yield trivial definite-valued observables locally. Obviously instability under decoherence is no longer be relevant.

Perhaps one could try to bypass Proposition 1 by exploiting extra structure not contained in the particular field state and local algebra to pick out the definite-valued observables in a region. For example, one might try to exploit the field's total energy-momentum operator, and, in particular, its generator of time evolution. In the context of the nonrelativistic modal interpretation, Bacciagaluppi *et al.* [19] (cf. also [15], p. 1181) have successfully invoked the analytic properties of the time evolution of the spectral projections of a system's reduced density operator D_S to avoid discontinuities that occur in the definite-valued set $\mathcal{M}_S$ at moments of time where the multiplicity of the eigenvalues of D_S changes. Their methods yield a natural dynamical way, independent of instability considerations, to avoid the trivial definite-valued sets determined by maximally mixed density operators. So one might hope that these same dynamical methods could be extended to type III$_1$ algebras so as to yield a richer set of properties in a local region than Proposition 1 allows for ergodic states. In any case, modal interpreters need to do more work to show that their interpretation yields sensible local properties in quantum field theory (even before one considers, with Dieks, how to define Lorentz invariant decoherent histories of properties).

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